Evolution of complex networks – how does the Internet grow

Final presentation

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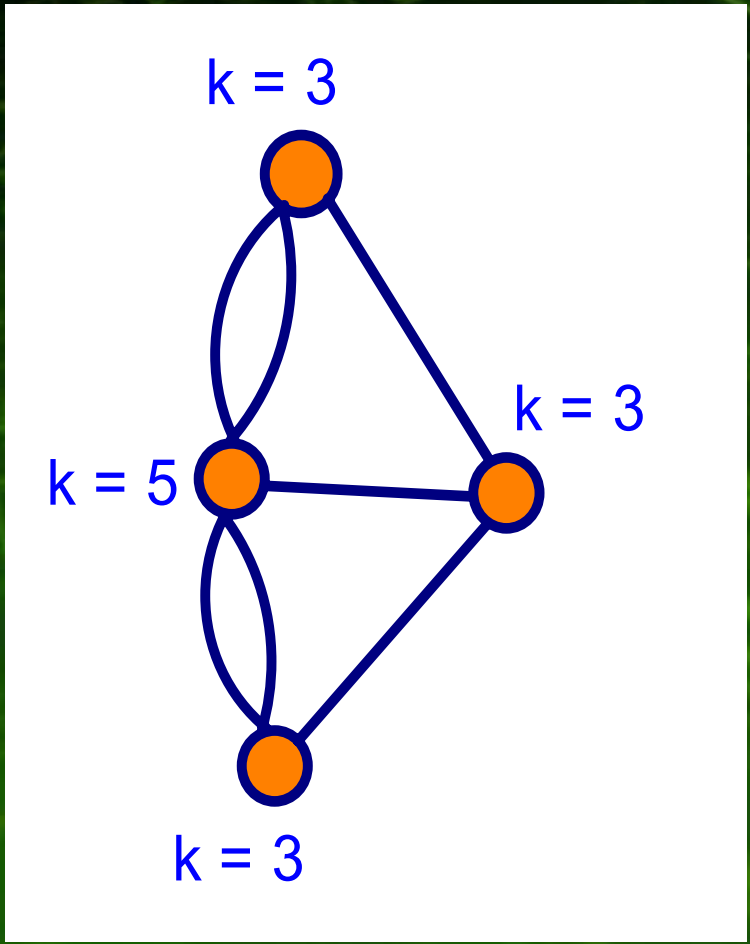
Complex networks

- between completely regular and completely random
- can describe very diverse systems
- also serve as a new way of representing complex systems
- main elements:
  - nodes
  - links



Description of complex networks

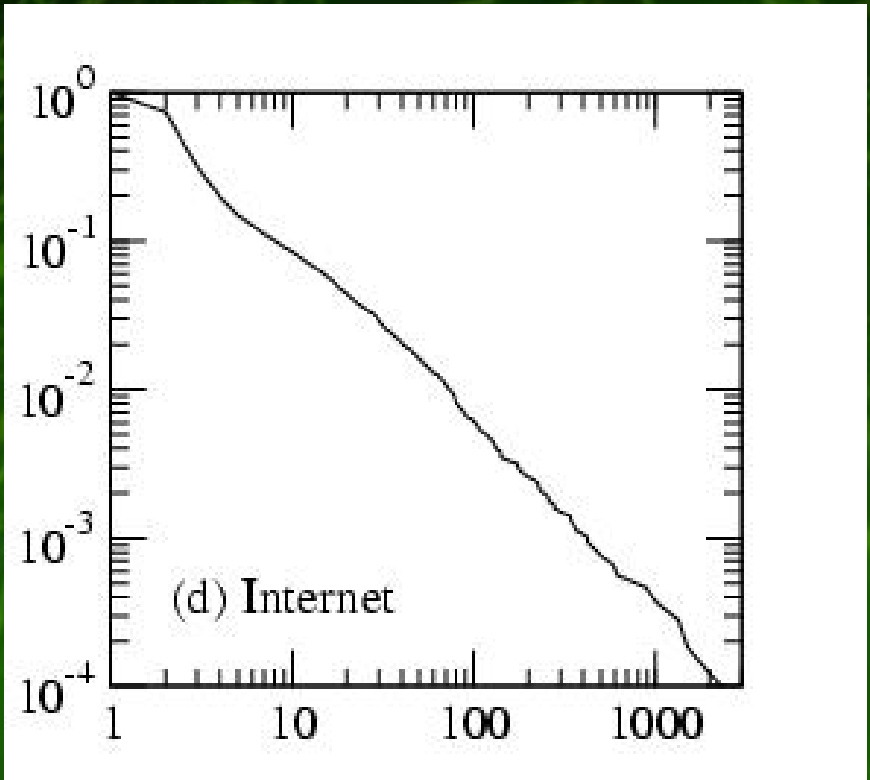
- node degree distribution
  - node degree k is the number of links attached to that node
  - node degree distribution P(k) is the probability that a randomly chosen node in a network has the degree k
- clustering coefficient
  - the ratio of the number of realized links between node's first neighbouring nodes and the maximal possible number of links between these nodes

$$C_i = \frac{2E_i}{k_i(k_i - 1)}$$

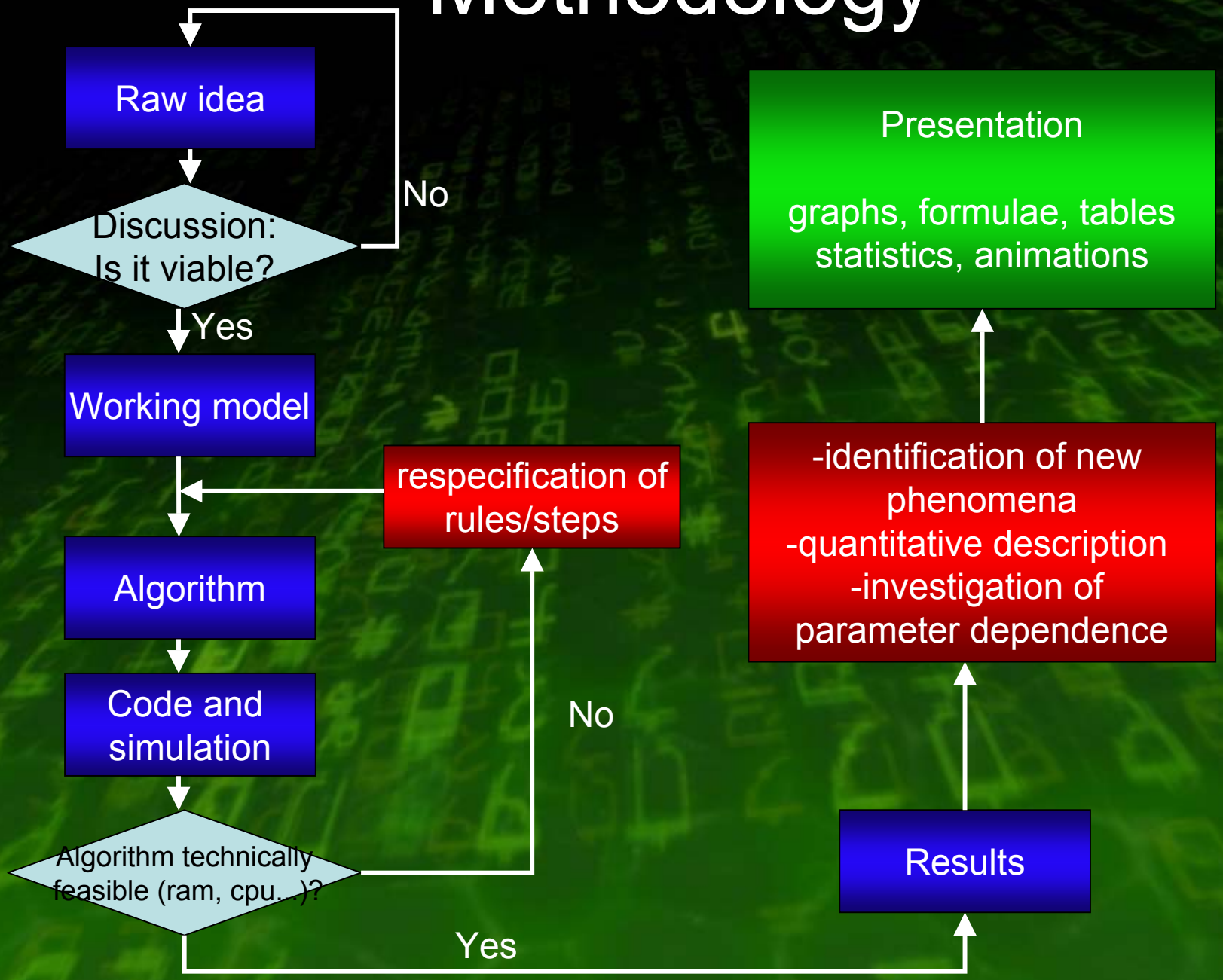
- average shortest path length

Complex networks in nature and society

- large majority caracterized by the straight line in the log k – log P(k) plot
- these are scale-free networks



Methodology



Goals of the complex networks study

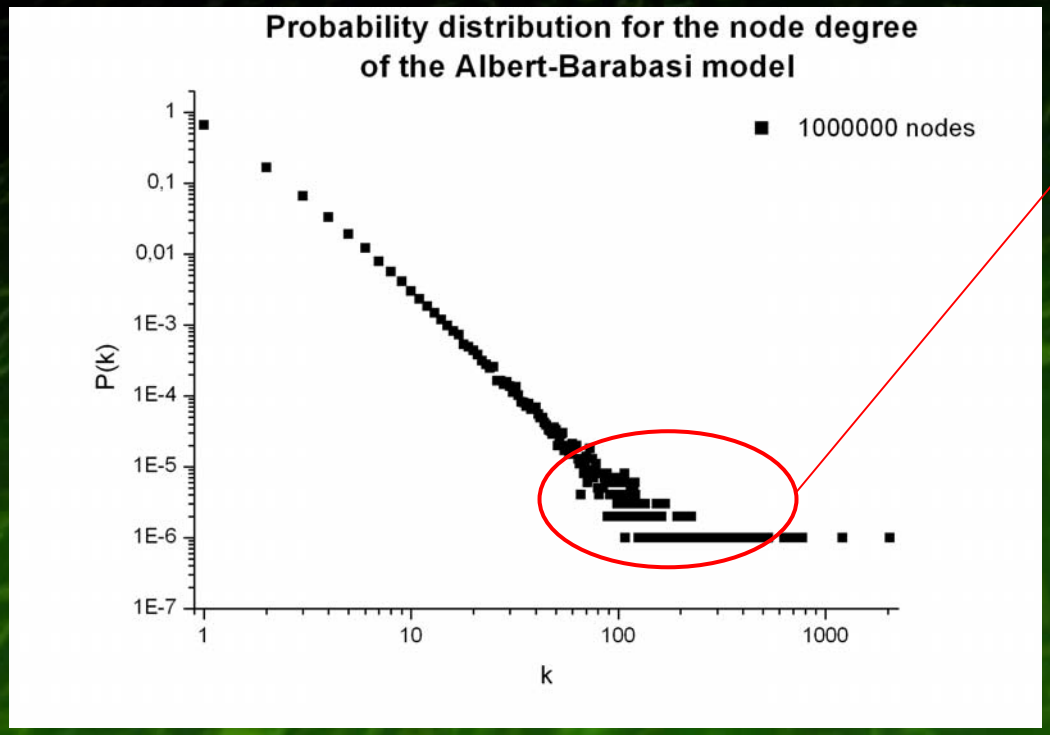
- getting familiar with the statistical description of complex networks
- studying the structure of complex networks by modeling their growth
- developing the software for the analysis of complex networks
- application to technological and/or sociological and/or biological problems

Albert - Barabasi model

- growing network – at each time step one node is added and attached to one of the existing nodes
- the new node is attached preferentially – it is more likely to attach itself to a node with more connections
- probability of attachment is proportional to the degree of the old node

Evolution of the network

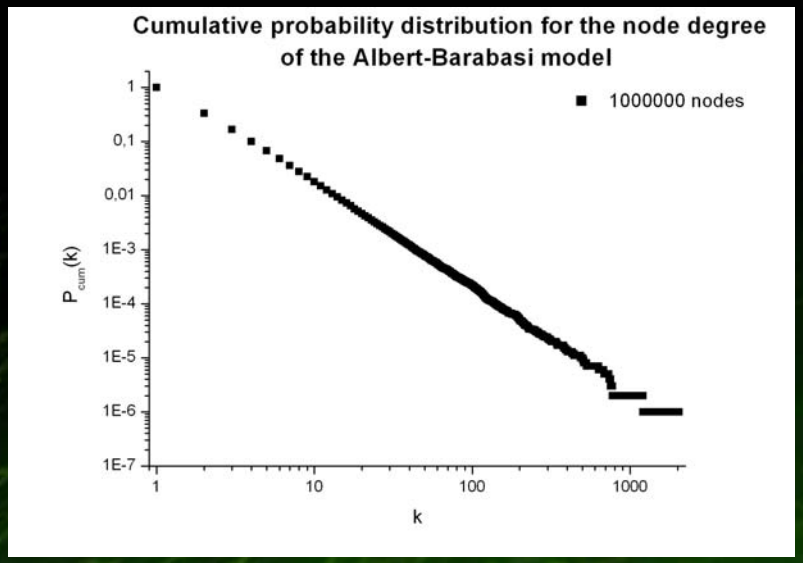
AB model results (1)

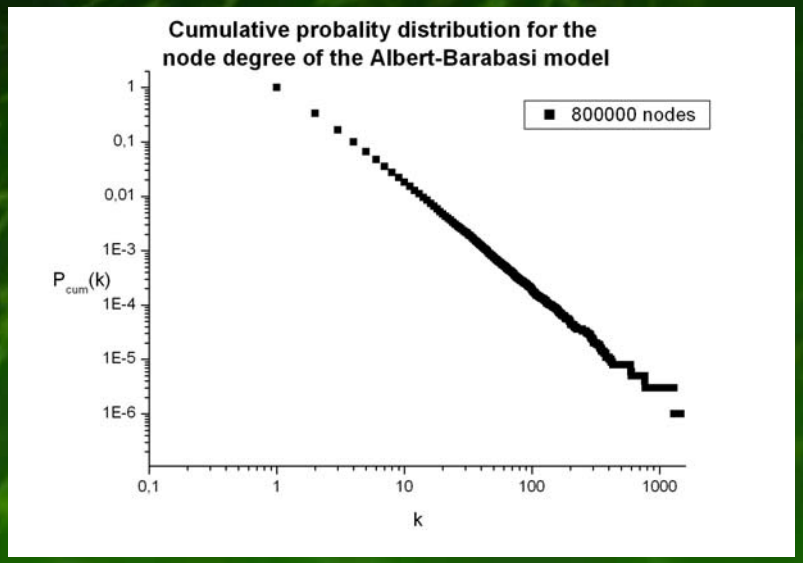


noise in the tail of the distribution

-straight line in the log-log plot
-slope is -3
- implies dependence $P(k) = Ck^{-3}$

AB model results (2)



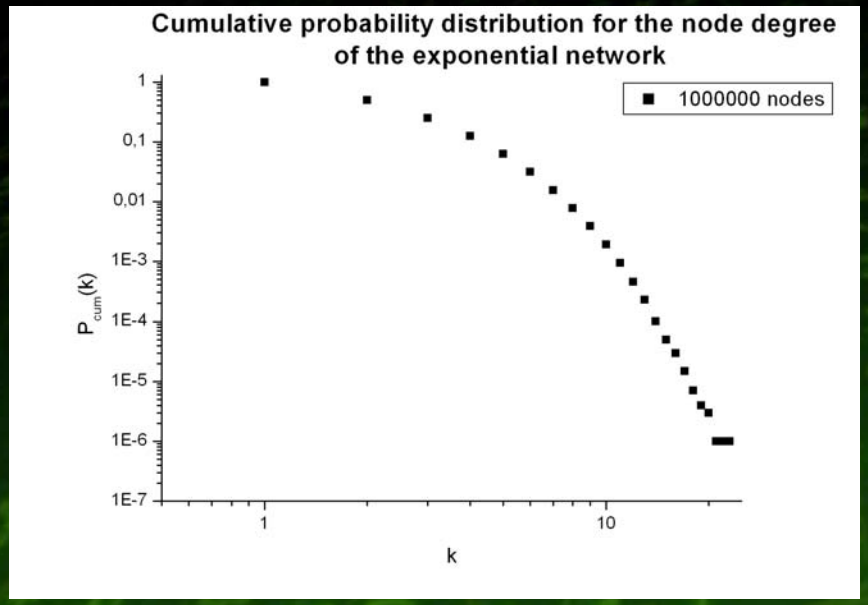


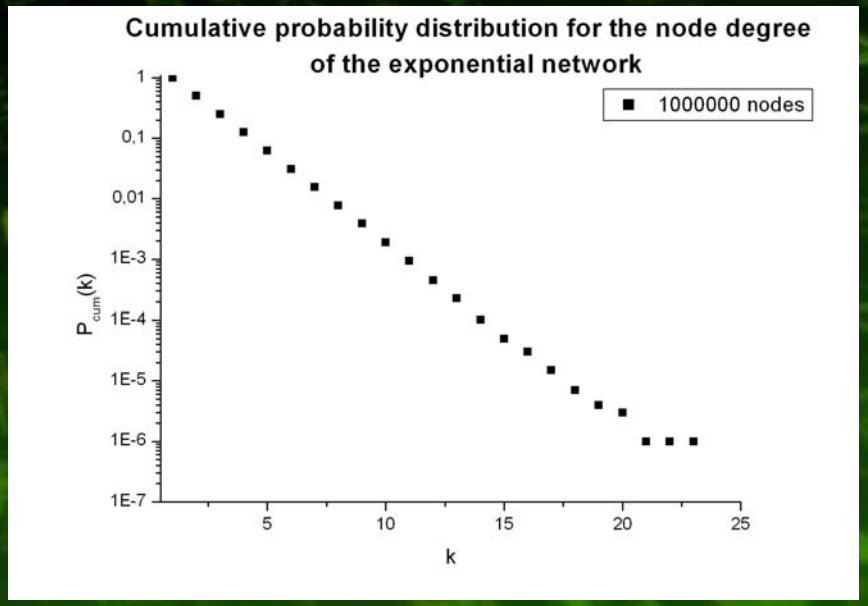
- **straight lines with slope -2**
- **noise significantly reduced**
- **dependence of the power law form**

$$P_{cum}(k)=Ak^{-2}$$

Agreement with the results of Barabasi and Albert

Testing of alternatives (random attachment)





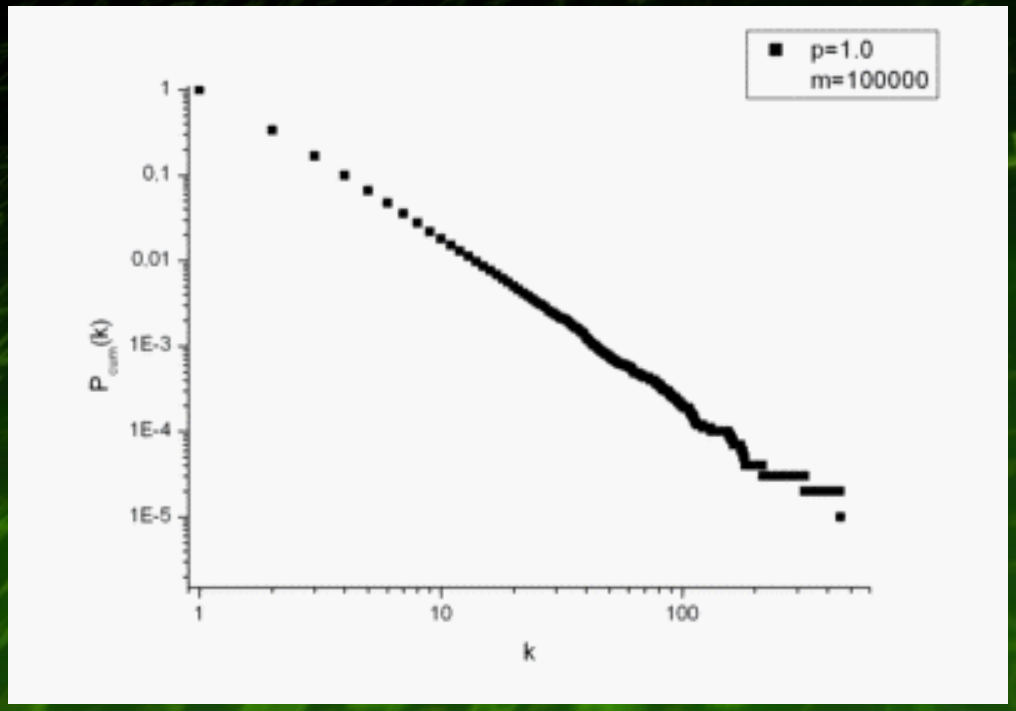
- no straight line in log-log plot

- straight line in lin-log plot

Exponential distribution → the network is not scale-free

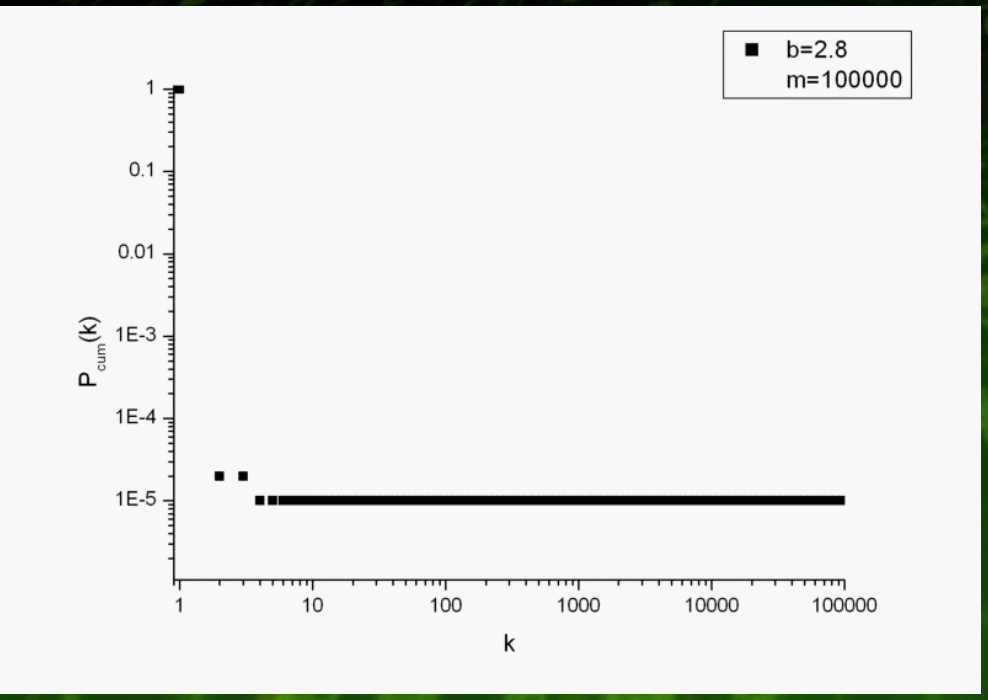
Conclusion: Preferential attachment is needed to obtain scale-free networks.

Hybrid network building



- combining exponential and power law networks
- randomly choosing between Albert-Barabasi model (with probability p) and random model (with probability 1-p) for the connection of the new node
- p=0 -> random model
- p=1 -> Albert-Barabasi model

Generalized preferential linking



- new parametar b introduced
- the new node is attached to the old node with probability ~ k^b (k is the degree of the old node)
- b=1 -> Albert-Barabasi model
- b=0 -> Random model
- b>1 -> condensation of links to a single node
- statistics of condensation:

  out of 30 simulations with 10000 nodes all contained a node with more than 9500 links

Adjacency matrix

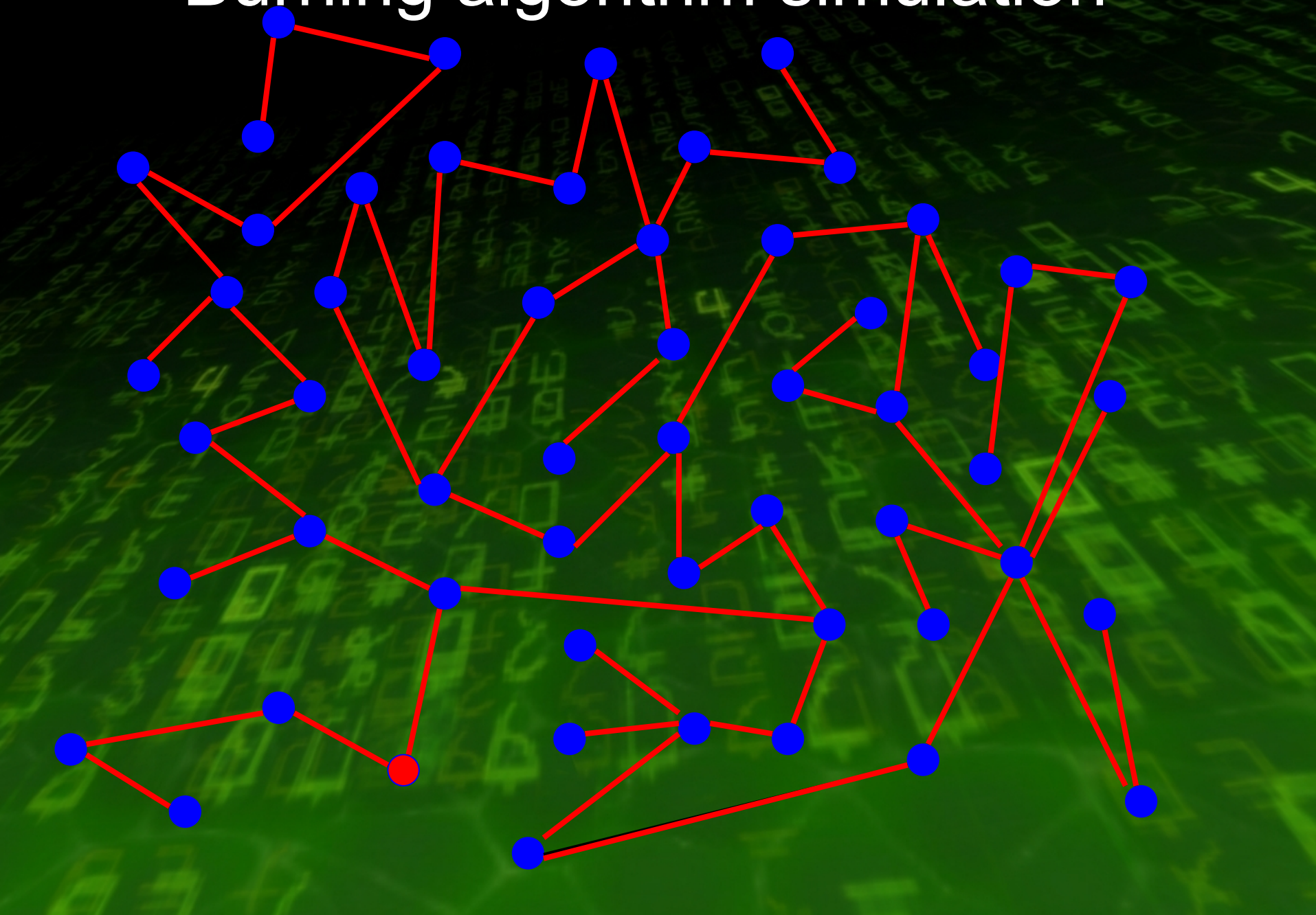
$$C = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

- Complete representation of a network
- $C_{ij} = 1$ if nodes i and j are connected
- $C_{ij} = 0$ if there is no connection between nodes i and j
- $C_{ij} = C_{ji}$
- $C_{ii} = 0$

Burning algorithm

- can be used to determine individual components of a complex network and to calculate the average shortest path
- functions like spreading of fire through the network
- each node that is on fire spreads the fire to its first neighbours if they are not already on fire

Burning algorithm simulation



Optimization of a network

- in order to connect componets of a network in the most efficient way (to increase the efficiency of a network) we first need to identify the components
- calculate its average shortest path
- by knowing it we can optimize a network with minimal cost (minimum number of new links)
- by burning algorithm calculate the best way to connect the components in order to decrease the average shortest path to minimum possible
- the power of the method becomes especially clear when connecting large components of a network

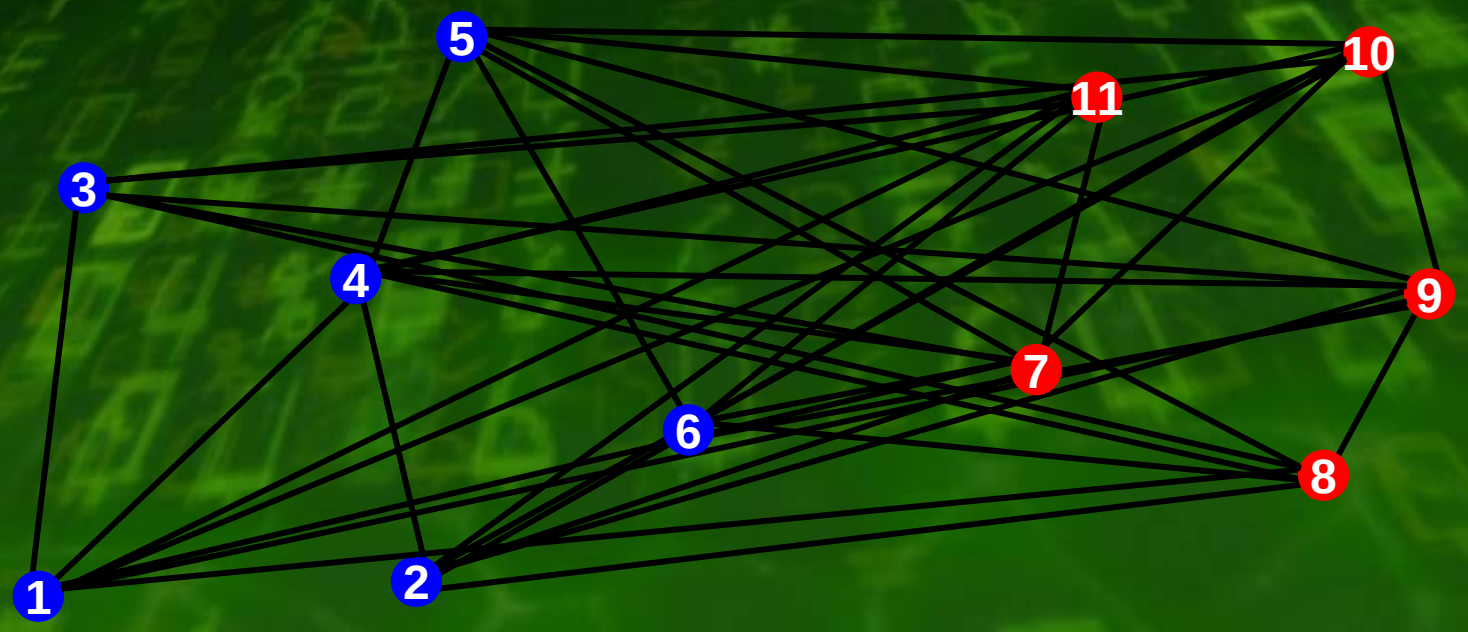
Two component optimization

- calculate the average shortest path before optimization
- a node from the first component connected to a node from the second component and the average shortest path of the so formed network calculated
- procedure repeated for all such pairs of nodes
- the combination of a node from the first and the second component with the minimum average shortest path is used as best optimization of a network using the addition of one link

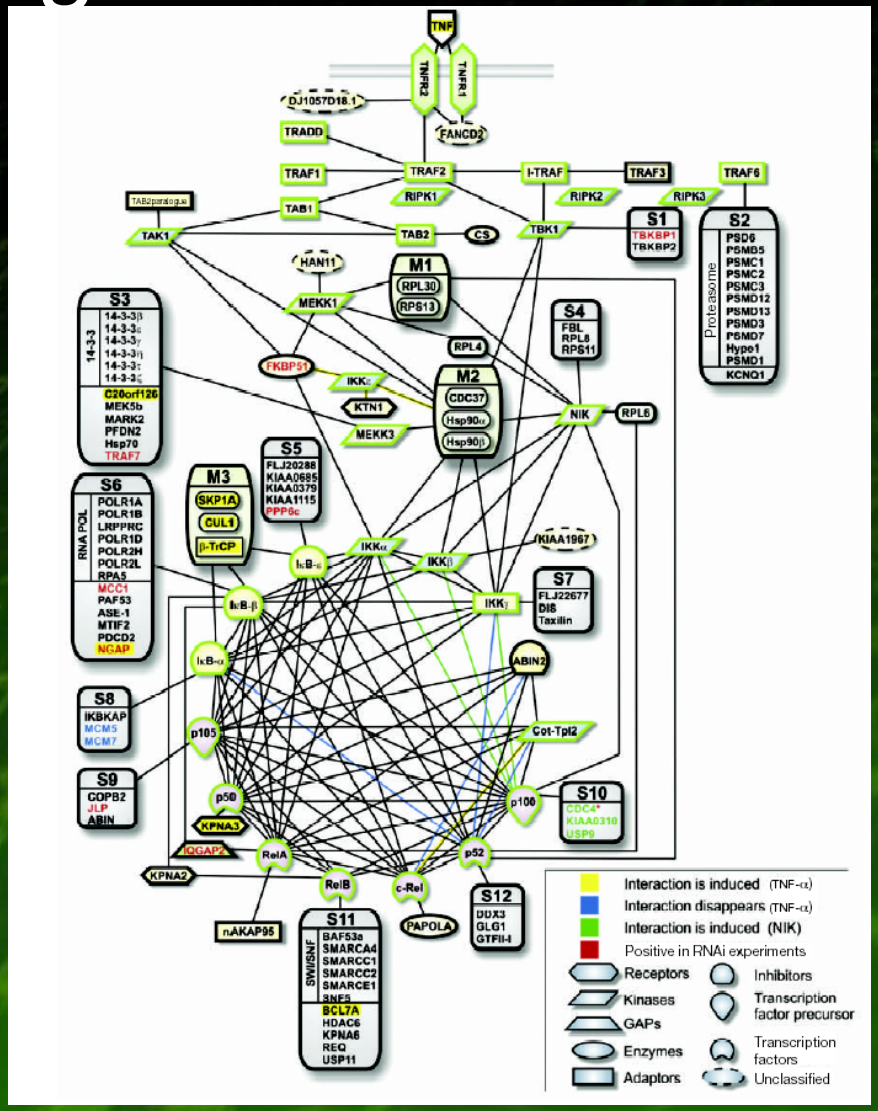
Before optimization: 3, 40

After optimization: 2, 091

Current Average Shortest Path 2, 330

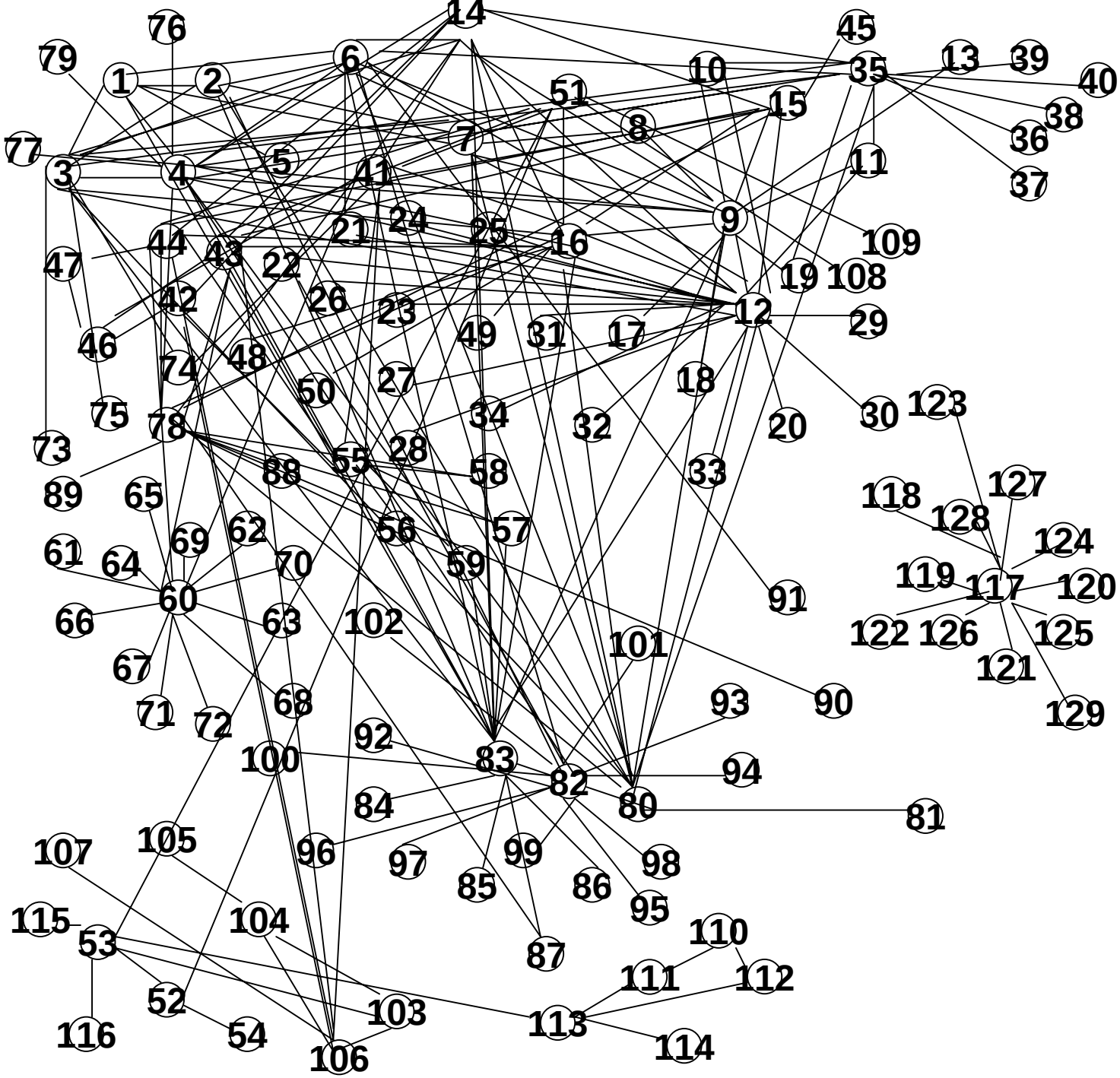


Analysis of the human TNF-α/NF-κB signal transduction pathway network (1)



- nodes – proteins
- links – protein interactions
- methods:
  - degree distribution
  - knocking out single nodes and looking at how that affects the number of components and the average shortest path
- T. Bouwmeester et al., Nature Cell Biol. **6,** 97-105 (2004).

In collaboration with the molecular biology group (Danijel)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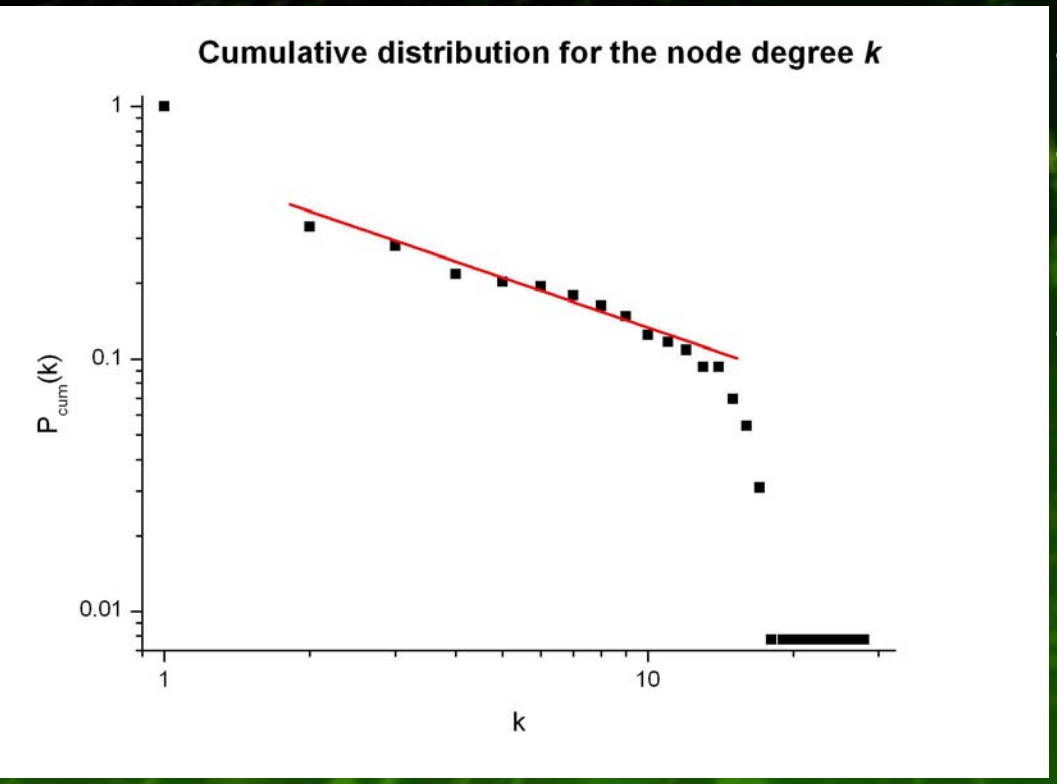
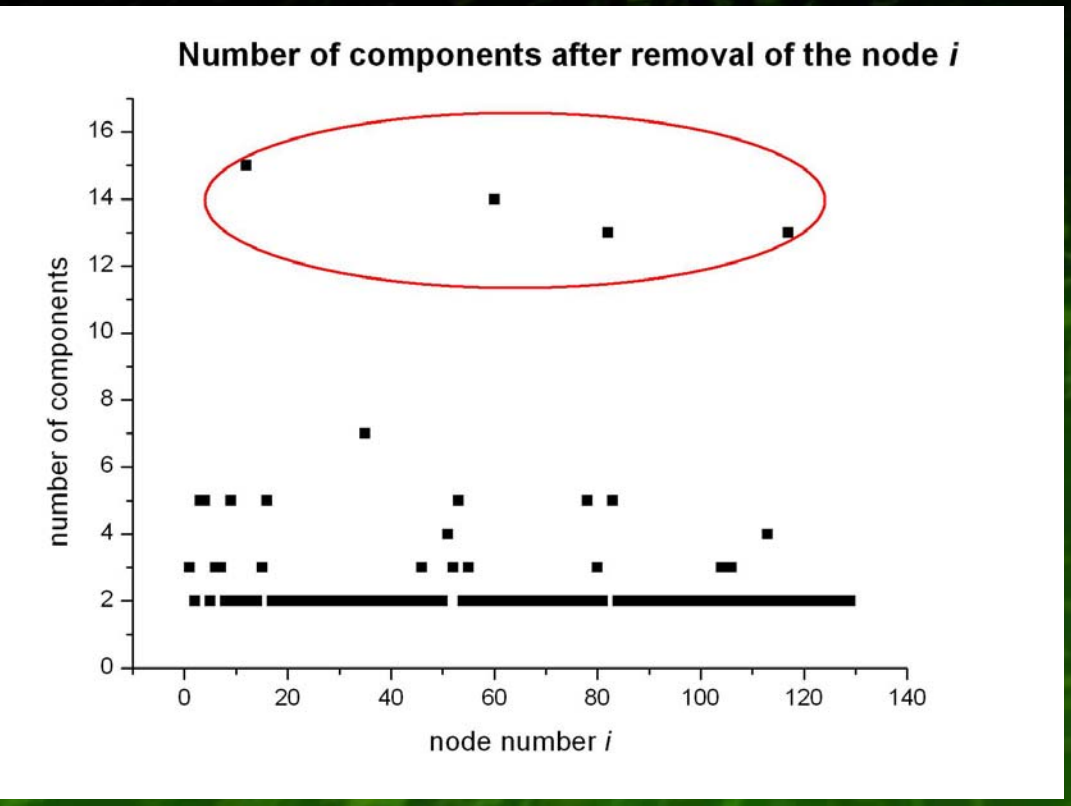
Analysis of the human TNF-α/NF-κB signal transduction pathway network (2)



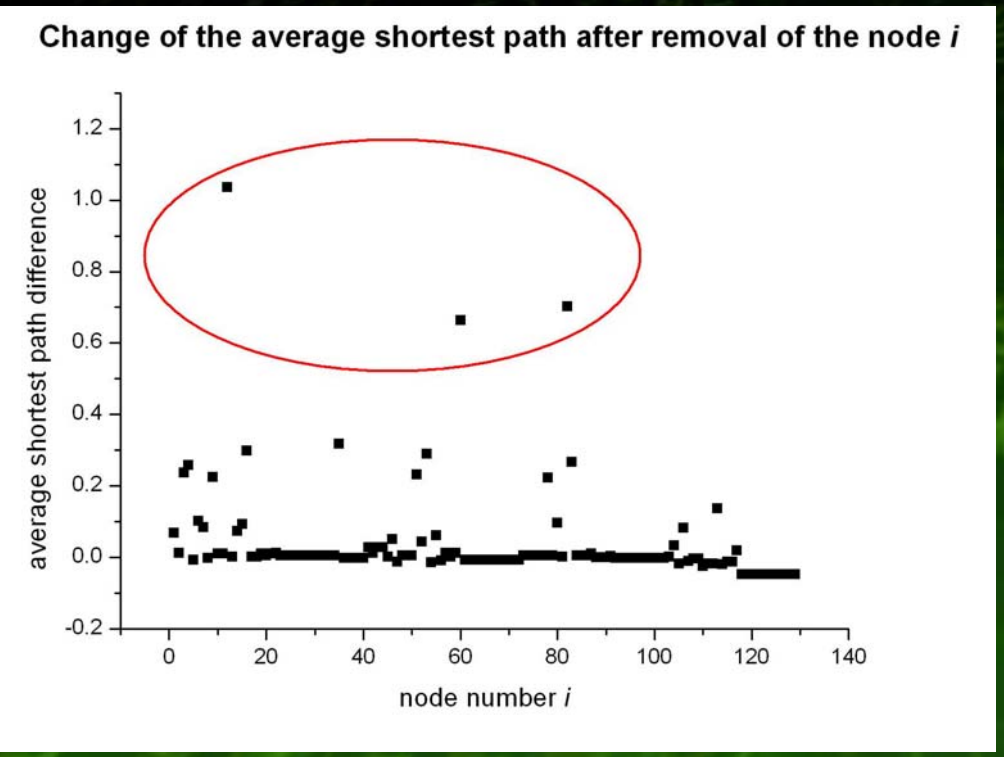
- cumulative distribution
- small network – limited statistics
- straight line in log k - log $P_{cum}(k)$ plot
  - indication that the network is scale-free
  - real network
  - sharp cutoff in the tail due to limited number of nodes

Analysis of the human TNF-α/NF-κB signal transduction pathway network (3)



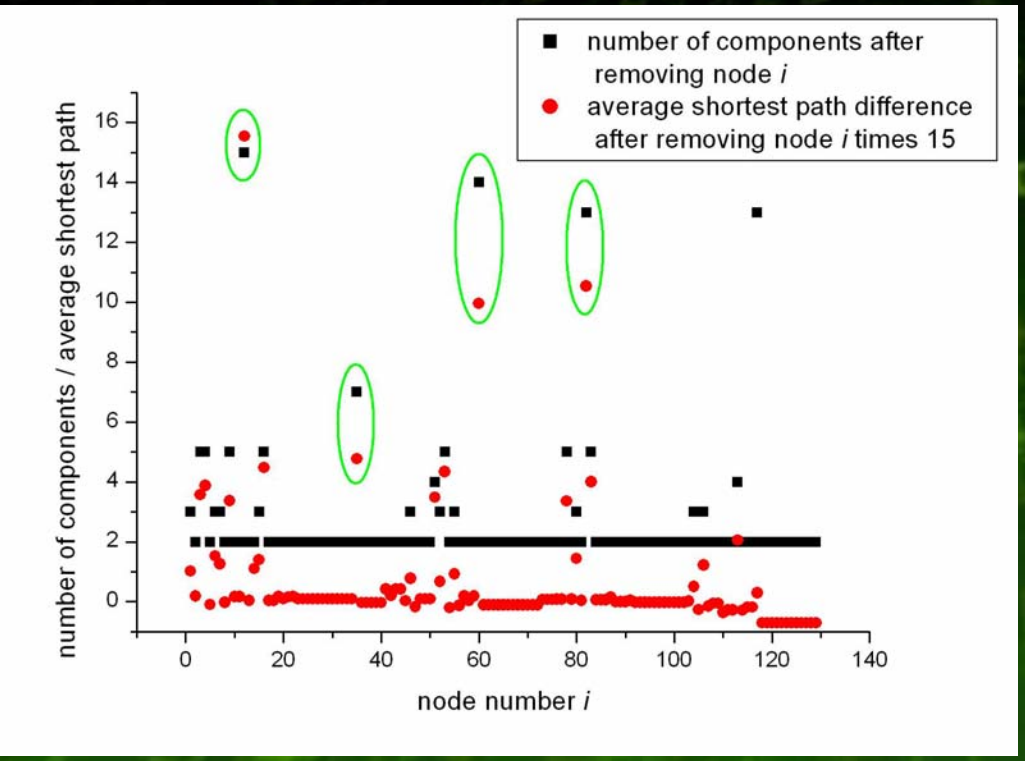
- sensitive to only certain nodes being removed – terorist attacks
- insensitive to removal of a large majority of nodes – malfunctions

Analysis of the human TNF-α/NF-κB signal transduction pathway network (4)



- the average shortest path also increases with removal of certain nodes

Analysis of the human TNF-α/NF-κB signal transduction pathway network (5)



- number of components and average shortest path after removing a node combined
- nodes that produce large difference in number of components also increase the average shortest path the most and vice versa

Our achievements

- all numerical simulations performed by the software developed by the team members
- all graphical simulations made by the team members
- real biological network completely analyzed by the software developed during the project

Hacker Team